

The Timing of Environmental Policies in the Presence of Extreme Events

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ABSTRACT

The recent applications of real options theory in environmental policy issues have illustrated the importance of explicitly modeling uncertainty and irreversibility in this type of problems. Within this framework, this paper explores the optimal timing of environmental policies when either the stock of pollutant or future economic costs caused by climate changes, are subject to normal as well as irregular changes. We assume that the stochastic evolution of these state variables is well described by a jump diffusion process and we examine alterations in optimal policies induced by the presence of discontinuities.

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1. Introduction

In modern developed societies, environmental issues are considered as of utmost priority. It is widely argued that the threat of environmental damages has mainly aroused due to human activities¹. One of these threats is caused by the concentration of harmful substances in the atmosphere, such as CO₂ gases, which are held responsible for the green house effect. According to the United Nations, the impacts of global climate change include: increase in global warming of about 1.4 to 5.8°C between 1990 and 2100 as predicted by current climate models, increase in the sea level of 9 - 88 cm by the year 2100, causing flooding of low-lying areas, increase in the magnitude of droughts and floods. As a result lives and livelihoods of human populations in coastal areas, arid and semi-arid areas, and cyclone-prone regions will be particularly at risk and damages from unpredictable storms and extreme weather conditions will constitute high costs to society (source: UNFCCC).

In recognition of this emerging environmental problem, governments have tried to coordinate on a global level in order to reduce the rate of emissions and minimize potential risks. The United Nations began a series of multilateral discussions in the late 1980s and early 1990s to address climate change. In 1992, these discussions culminated in the United Nations Framework Convention on Climate Change (UNFCCC), which was signed by 154 parties (or states). As part of its mandate, the UNFCCC initiated a negotiation process among nations seeking to solidify commitments to reduce the impacts of climate change. The negotiations culminated with the adoption of the Kyoto Protocol in December 1997. This phenomenon of

¹ There are scientists who do not embrace this explanation. They claim, for instance, that the increase in the earth's temperature is a natural phenomenon that repeats itself in the course of centuries.

societies manufacturing and managing risk can well be described by what the prominent sociologist Ulrich Beck (1992) has termed as “reflexive modernity”.

Interestingly enough, in the last years has emerged a growing literature that applies optimal timing problems in environmental issues. More specifically, the question that is often addressed refers to the optimal point where societies should adopt a costly policy to reduce emissions of some environmental pollutant. Hitherto, the analysis of optimal environmental policies has been examined either by means of traditional cost-benefit analysis, or from the perspective of modern real options theory. The latter approach is superior to the former, since it takes explicitly into account three important characteristics of environmental problems, namely uncertainty, irreversibility and sunk costs and benefits.

Within the framework of real options theory, and in particular to that proposed by Pindyck (2000, 2002), this paper examines the characteristics of optimal timing decisions related to the adoption of an environmental policy when there is a possibility of abnormal changes either in the evolution of the stock of pollutant, or in the future costs and benefits of environmental damage and its reduction. For example, large changes in the concentration of GHG, such as CO₂ gases, may appear during extreme winter or summers because of an increase in electricity consumption. On a regional basis, major accidents (e.g., fires in oil wells) could be held responsible for a sudden increase in the rate of CO₂ emissions. Regarding economic uncertainty,

temperature increase may cause sudden changes in earth's climate with serious consequences on natural resources, agricultural output and land use².

To further support our hypothesis that the stock of pollutant can be subject to large changes we refer to the following incident. On October 11, 2004 the Independent reported that the atmospheric levels of carbon dioxide (CO₂), the principal greenhouse gas, made a sudden jump that cannot be explained by any corresponding jump in terrestrial emissions of CO₂ from power stations and motor vehicles—because there has been none. It was the first time the quantity of CO₂ in the atmosphere had risen by more than two parts per million over two consecutive years. Some scientists believe the abrupt rise may be evidence of the climate change "feedback" mechanism, by which global warming alters the earth's natural systems causing warming to increase even faster than before, according to the report. The average rise in CO₂ levels has been about 1.6 parts per million by volume in recent decades, although there have been several peaks associated with El Nino, a disruptive weather pattern in the tropical Pacific. However, in the last two years the level has risen by 2.08ppm and 2.54ppm and neither were El Nino years.

Other studies³ that apply real options methodology in continuous time include Saphores and Carr (2000), where they derive optimal policy rules when the stock of pollutant follows a square root mean reverting process. Hendricks (1992) develops a continuous-time model of global warming, similar to Pindyck (2002), and he examines the effects of learning when global mean temperature is linked to the

² As mentioned also by Fisher and Narain (2003), others claim that the economic consequences will be modest, or even that the net impact of warming will be beneficial.

³ For comprehensive literature reviews see Pindyck (2000) and Fisher and Narain (2003).

atmospheric GHG concentration. Finally, Conrad (1992) applies a model where the social cost of pollution and the stock of pollutant are connected via a parameter that evolves as a geometric Brownian motion.

This paper is structured as follows. Section 2 examines optimal policies when the evolution of the stock of pollutant exhibits discontinuities; section 3 extends the analysis for the case where social cost per unit of stock of pollutant is allowed to rise sharply and finally section 4 concludes.

2. Environmental Uncertainty

We assume that the stock of pollutant, denoted by M_t , follows a gaussian mean reverting process augmented by jumps:

$$dM_t = [\beta E(t) - \delta M(t)]dt + \sigma dZ + \phi dq \quad (1)$$

The jump part is controlled by compound Poisson process with parameter λ , and positive jump size ϕ . The jump size can be constant or drawn from a well-behaved distribution (e.g., exponential). For reasons of tractability we let ϕ to be a constant. The parameter $E(t)$ is a flow variable that controls M_t , e.g., the rate of CO₂ emissions and the parameter δ stands for the natural rate at which the stock of pollutant decays over time. We assume that the flow of social cost associated with M_t is $B(M_t, \theta_t)$ and more specifically that $B(M_t, \theta_t)$, is convex in M , for example $B(M_t, \theta_t) = -\theta M^2$. The parameter θ indicates the unit damage cost and is assumed to be constant. We do not examine linear forms of benefit functions since in such a case the optimal policy is independent from the evolution of the stock of pollutant (see Pindyck, 2000). We

assume that by incurring a fixed cost K , the rate of emissions drops to a new value E^* and λ becomes zero. For simplicity, we set the sunk cost of an emission reduction, for both the diffusion and jump part, to be linear in the size of the reduction. Furthermore, we assume that emissions reduce to zero once a policy is adopted. Thus, the cost of policy adoption to reduce emissions to zero is $K_1 = mE_0$ and $E^* = 0$. In addition to that, the cost of policy adoption to reduce λ to zero is $K_2 = n\lambda$. Therefore, the total cost is $K = K_1 + K_2$.

The Bellman equations for the value functions W^N and W^A , in the *no-adopt* and *adopt* regions, respectively are:

$$[\beta E - \delta M]W_M^N + \frac{1}{2}\sigma^2 W_{MM}^N - (r + \lambda)W^N + \lambda W^N(M + \phi) = \theta M^2 \quad (2)$$

$$[\beta E^* - \delta M]W_M^A + \frac{1}{2}\sigma^2 W_{MM}^A - rW^A = \theta M^2 \quad (3)$$

The value functions must satisfy the following boundary conditions:

$$W_M^A(0) = 0 \quad (4)$$

$$W^N(M^*) = W^A(M^*) - K \quad (5)$$

$$W_N^M(M^*) = W_N^A(M^*) \quad (6)$$

where M^* is the critical value of M where policy should be applied.

Following Pindyck (2000), the first condition just says that when M becomes zero, the value function at the adopt region should reach its maximum. The second condition is the value-matching; it says that when M reaches the critical level M^* and society exercises its option to adopt the policy by incurring a sunk cost K , it receives the net

payoff $W^A(M^*) - K$. The third condition is the smooth-pasting; if adoption at M^* is indeed optimal, the derivative of the value function must be continuous at M^* .

When $\delta > 0$, then the value function in the “no adopt area” can only be solved numerically. In order to obtain a closed form solution we assume that $\delta = 0$, that is, the environmental damage is completely irreversible. Under this assumption the value function in the no adopt area is given by:

$$\beta E W_M^N + \frac{1}{2} \sigma^2 W_{MM}^N - (r + \lambda) W^N + \lambda W^N (M + \phi) = \theta M^2 \quad (7)$$

The general solution to the above ODE is:

$$W = C_1 e^{k_1 M} + C_2 e^{k_2 M} \quad (8)$$

where k_1 and k_2 satisfy the following characteristic equation:

$$\frac{1}{2} \sigma^2 k^2 + \beta E k - (\lambda + r) + \lambda e^{k\phi} = 0 \quad (9)$$

Note that this equation can only be solved numerically. The particular solution of (7) is:

$$W_p = -\frac{(\sigma^2 + \lambda\phi^2 + rM^2)\theta}{r^2} - \frac{2\theta(\beta E + \lambda\phi)(\beta E + \lambda\phi + rM)}{r^3} \quad (10)$$

and thus the solution in the “no adopt region” is given by:

$$W^N = C_1 e^{k_1 M} + C_2 e^{k_2 M} - \frac{(\sigma^2 + \lambda \phi^2 + rM^2)\theta}{r^2} - \frac{2\theta\beta E(\beta E + \lambda\phi + rM)}{r^3} - \frac{2\theta\lambda\phi(\beta E + \lambda\phi + rM)}{r^3} \quad (11)$$

The first two terms in equation (11) correspond to the value of the option to adopt the policy. The third term is the present value of the flow of social cost from the current stock of the pollutant, M , allowing M to evolve stochastically in the future. The fourth term stands for the present value of the flow of social cost, given that the emissions continued to evolve perpetually at a rate E . The fifth term is the present value of the flow of social cost, given that there is probability λ that the stock of pollutant will increase by ϕ .

As previously mentioned, the first two terms on the right-hand side of equation (11) are the value of the option to adopt the policy. With k_2 negative, as M tends to infinity, $e^{k_2 M}$ tends to zero. However, it is impossible that the value of the option to adopt the policy becomes zero as M tends to infinity. Consequently, C_2 should be zero.

Therefore we obtain:

$$W^N(M) = C_1 e^{k_1 M} - \frac{(\sigma^2 + \lambda \phi^2 + rM^2)\theta}{r^2} - \frac{2\theta(\beta E + \lambda\phi)(\beta E + \lambda\phi + rM)}{r^3} \quad (12)$$

The value function in the adopt region is given by:

$$\frac{1}{2}\sigma^2 W_{MM}^A - rW^A = \theta M^2 \quad (13)$$

with the following solution:

$$W^A(M) = -\frac{\theta M^2}{r} - \frac{\sigma^2 \theta}{r^2} \quad (14)$$

Given k_1 , C_1 and M^* can be found from the boundary conditions (5), (6) and the value function in the adopt area.

From the smooth-pasting condition, we obtain:

$$C_1 = \frac{\frac{2}{r^2}(\beta E + \lambda \phi)\theta}{k_1 e^{k_1 M^*}}$$

Substituting this expression for C_1 into the value-matching condition, gives:

$$\frac{2}{r^2} \frac{(\beta E + \lambda \phi)\theta}{k_1 e^{k_1 M^*}} - \frac{2}{r^3} (\beta E + \lambda \phi)^2 \theta - \frac{(\sigma^2 + \lambda \phi^2)\theta}{r^2} + \frac{\sigma^2 \theta}{r^2} + K = \frac{2}{r^2} \theta M^* (\beta E + \lambda \phi)$$

Solving for M^* , we obtain:

$$M^* = \frac{1}{k_1} - \frac{1}{r} (\beta E_0 + \lambda \phi) - \frac{\lambda \phi^2}{2(\beta E_0 + \lambda \phi)} + \frac{Kr^2}{2\theta(\beta E_0 + \lambda \phi)} \quad (15)$$

For λ equal to zero equation (15) collapses to that of Pindyck (2000). As expected, the critical value M^* is a decreasing function of both λ and ϕ . If the stock of pollutant is subject to large unexpected changes then policy adoption should be undertaken earlier. Furthermore, we observe that when we let the probability of a jump and/or the magnitude of the jump size to grow arbitrarily, then policy should be adopted immediately. For illustrative purposes we consider a numerical example, using for the continuous part the same values as those in Pindyck (2000). We set $r=0.04$, $K=4$, $E_0=0.3$, $\beta=1$, $\theta=0.002$, $\lambda=0.1$ and $\phi=0.07$. The last two parameters stand for the jump part and can be interpreted as a probability of jump every 10 years that causes a 7% increase in the stock of pollutant. For $\sigma=1$ and $\sigma=4$, Pindyck (2000) finds that policy

should be adopted when $M \geq M^* = 6.74$ and $M \geq M^* = 16.21$, respectively. In the presence of jumps, policy should be adopted earlier when $M^* = 6.59$ and $M^* = 16.02$. For the more general case of $\delta > 0$ we can safely conclude that the level of M^* , where policy should be adopted, will again be inversely related to parameters controlling the discontinuous part.

So far we have considered the scenario where by adopting an environmental policy the probability of a jump drops to zero. However, this might be unrealistic so we will investigate two more general cases. In the first one, we assume that by incurring a fixed cost K , the rate of emissions drops again to zero but now λ drops to a new value λ^* , greater than zero. The Bellman equations for the value functions W^N and W^A , in the *no-adopt* and *adopt* regions, respectively are:

$$\beta E W_M^N + \frac{1}{2} \sigma^2 W_{MM}^N - (r + \lambda) W^N + \lambda W^N (M + \phi) = \theta M^2$$

$$\frac{1}{2} \sigma^2 W_{MM}^A - (r + \lambda^*) W^A + \lambda^* W^A (M + \phi) = \theta M^2$$

The value functions must satisfy the same boundary conditions. The solution in the “no adopt region” is given by (12).

The solution of the value function in the adopt region is given by:

$$W^A(M) = -\frac{(\sigma^2 + \phi^2 + rM^2)\theta}{r^2} - \frac{2\theta\lambda^*\phi(\lambda^*\phi + rM)}{r^3} \quad (16)$$

The first term is the present value of the flow of social cost from the current stock of the pollutant, M , whereas the second term is the present value of the flow of social cost, given that there is probability λ^* that the stock of pollutant will increase by ϕ .

Given k_1 , C_1 and M^* can be found from the boundary conditions (5), (6) and the value function in the adopt area. From the smooth-pasting condition, we obtain:

$$C_1 = \frac{\frac{2}{r^2} [\beta E + (\lambda - \lambda^*) \phi] \theta}{k_1 e^{k_1 M^*}}$$

Substituting this expression for C_1 into the value-matching condition and solving for M^* , we obtain:

$$M^* = \frac{1}{k_1} + \frac{Kr^2}{2\theta [\beta E_0 + (\lambda - \lambda^*) \phi]} - \frac{1}{r} \frac{\beta^2 E_0^2 + \phi^2 (\lambda^2 - \lambda^{*2}) + 2\beta E_0 \lambda \phi}{[\beta E_0 + (\lambda - \lambda^*) \phi]} - \frac{(\lambda - \lambda^*) \phi^2}{2 [\beta E_0 + (\lambda - \lambda^*) \phi]} \quad (17)$$

Note that when $\lambda^* = 0$, the above equation coincides with equation (15). In the second case, we assume that there is no change in the probability of a jump occurring, i.e. $\lambda^* = \lambda$. Therefore, we obtain:

$$M^* = \frac{1}{k_1} + \frac{Kr^2}{2\theta \beta E_0} - \frac{1}{r} \beta E_0 \quad (18)$$

3. Economic Uncertainty with Jumps

In this section we introduce economic uncertainty, denoted by θ , and we assume that is governed by a geometric Brownian motion augmented by jumps

$$d\theta = \alpha \theta dt + \sigma \theta dZ + \phi \theta dq \quad (19)$$

Ecological uncertainty now follows a deterministic differential equation of the following form

$$dM(t)/dt = \beta E(t) - \delta M(t) \quad (20)$$

As in the case of environmental uncertainty, we assume that the jump part is controlled by a Poisson process with positive constant jump sizes. As in Pindyck (2000) we assume that the flow of social cost is a linear function of M and that the policy adoption implies reducing E from the initial level to zero with a cost $K=kE_0$. Furthermore, we assume that the discontinuous part of the economic costs remains even after allowing for policy adoption.

The value functions for the “no-adopt” ($E_t=E_0$) and the “adopt” region ($E_t=0$) respectively, must satisfy the following Bellman equations:

$$rW^N = -\theta M + (\beta E_0 - \delta M)W_M^N + \alpha \theta W_\theta^N + \frac{1}{2} \sigma^2 \theta W_{\theta\theta}^N - \lambda W^N(\theta, M) + \lambda W^N(\theta(\phi+1), M) \quad (21)$$

$$rW^A = -\theta M - \delta M W_M^A + \alpha \theta W_\theta^A + \frac{1}{2} \sigma^2 \theta W_{\theta\theta}^A - \lambda W^A(\theta, M) + \lambda M W^A(\theta(\phi+1), M) \quad (22)$$

subject to the boundary conditions:

$$\begin{aligned} W^N(0, M) &= 0 \\ W^N(\theta^*, M) &= W^A(\theta^*, M) - K \\ W_\theta^N(\theta^*, M) &= W_\theta^A(\theta^*, M) \end{aligned} \quad (23)$$

The first boundary condition states that zero is an absorbing barrier for θ , the second boundary is the value matching condition and the third the smooth pasting condition.

The solution for the *no adopt* area is given by:

$$W^N(\theta, M) = A\theta^\gamma - \frac{\theta M}{r + \delta - (\alpha + \lambda\phi)} - \frac{\beta E_0 \theta}{(r - (\alpha + \lambda\phi))(r + \delta - (\alpha + \lambda\phi))} \quad (24)$$

where γ is the solution to the following characteristic equation:

$$\frac{1}{2}\sigma^2\gamma(\gamma-1) + \alpha\gamma - (r + \lambda) + \lambda(1 + \phi)^\gamma = 0 \quad (25)$$

The solution for the “adopt” area is given by

$$W^A(\theta, M) = -\frac{\theta M}{r + \delta - (\alpha + \lambda\phi)} \quad (26)$$

The first term in (24) is the value of the option and the second term stands for the present value of the social cost caused by the current cost of pollutant. Compared to the solution with no jumps ($\lambda=0$) we observe that the present value of social costs is larger due to the fact that now θ has an expected rate of growth which is the sum of two terms; the drift of the continuous part and the expected jump size $\lambda\phi$, of the discontinuous part. Because the jump sizes are positive, the discount rate will be smaller. Again, for the same reason, the present value of the flow of social costs if emissions continued forever at a rate of E_0 is larger.

From the boundary conditions, the constant A and the value of θ^* where policy should be adopted, are given by:

$$A = E_0 \left(\frac{\gamma - 1}{k} \right)^{\gamma - 1} \left[\frac{\beta}{(r - a - \lambda\phi)(r + \delta - \alpha - \lambda\phi)\gamma} \right]^\gamma \quad (27)$$

$$\theta^* = \left(\frac{\gamma}{\gamma - 1} \right) k (r - (a + \lambda\phi))(r + \delta - (\alpha + \lambda\phi)) / \beta \quad (28)$$

The term θ^* is a strictly decreasing function of both λ and ϕ . An increase in the probability of a large change in the future flow of social costs will cause environmental policies to be adopted earlier. An increase in the jump size will have a similar effect. In the case where the probability of the Poisson event and the size of the jump move to opposite directions, the change in θ^* will depend on the magnitude of their relative changes.

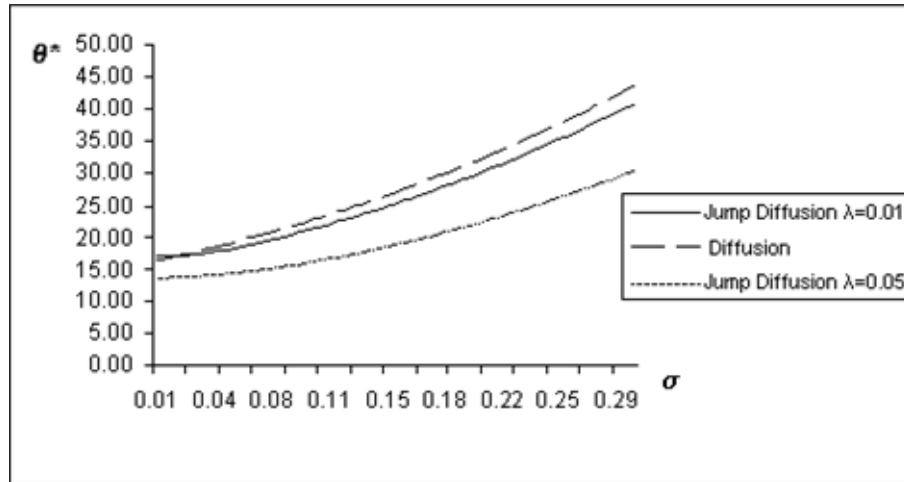


Figure 1: The optimal θ^* implied by the jump diffusion and the diffusion case, respectively.

We consider a numerical example, and for reasons of comparison we use the same parameters as in Pindyck (2000). We let $\alpha=0$, $r=0.04$, $\delta=0.02$, $\sigma=0.2$, $\beta=1$, $E_0=300,000$ tons/year, $\theta_0 = \$20$ per ton and $k=6667$ so that $K=kE_0=\$2$ billion. For the jump part we assume that $\lambda=0.01$ (jump every 100 years) and $\phi=0.3$. For this particular values,

policy should be undertaken when $\theta^* = 29.87$ as opposed to the diffusion case where the critical value is given by $\theta^* = 32$. As it is illustrated in Figure 1, an increase in the jump probability to $\lambda=0.05$ (jump event every 20 years) shifts the critical value θ^* to the right and therefore policy should be adopted earlier.

4. Conclusions

This paper has examined alternations in optimal policies induced by the presence of discontinuities either in the stock of pollutant or in the future economic costs caused by climate changes. We extended the analysis by Pindyck (2000) by allowing the state variables to follow jump diffusion processes. Sudden jumps in the concentration of CO₂ in the atmosphere could be justified by the climate change "feedback" mechanism, as explained earlier in this paper. Neglecting the occurrence of jumps in the stock of pollutant could be misleading as to the optimal timing of environmental policy adoption. As expected, we find that when the stock of pollutant is subject to large changes, policy should be undertaken earlier. Similar results are obtained when the social cost per unit of stock of pollutant is allowed to rise sharply. Overall, the presence of discontinuities can account for a large part of the optimal environmental policy. Further research should concentrate on trying to obtain closed form solutions when the probability of a jump in social costs depends on the level of the stock of pollutant. In such a case, the discontinuous part could be modeled as a Poisson process with time varying intensity (Cox process type). We leave this issue for future research.

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